

# Generation of entanglement via atomic coherence in a two-mode three-level cascade atomic system

Y.-X. Ping<sup>a</sup>, B. Zhang, and Z. Cheng

Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, P.R. China

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**Abstract.** The generation of continuous variable entanglement via atomic coherence in a two-mode three-level cascade atomic system is discussed according to the entanglement criterion proposed by Duan et al. [Phys. Rev. Lett. **84**, 2722 (2000)]. Atomic coherence between the top and bottom levels is induced with two photons of a strong external pump field. It shows that entanglement for the two-mode field in the cavity can be generated under certain conditions. Moreover, by means of the input-output theory, we show that the two-mode entanglement could also be approached at the output.

**PACS.** 42.50.Dv Nonclassical states of the electromagnetic field, including entangled photon states; quantum state engineering and measurements – 03.65.Ud Entanglement and quantum nonlocality (e.g. EPR paradox, Bell's inequalities, GHZ states, etc.)

**QICS.** 02.30.-n Entanglement, nonlocality, complementarity – 03.05.+c Characterization and classification of entanglement

## 1 Introduction

Entanglement is one of the most characteristic properties that makes quantum theory distinct from classical theory. It plays an essential role in quantum information processing such as quantum teleportation [1], quantum computation [2], and quantum cryptography [3]. In recent years, entanglement in continuous variable (CV) systems [4] has been attracted great interest both theoretically and experimentally. For instance, many criteria as the measure of entanglement have been proposed for CV systems [5–9]. In experiments, the first observation of quantum teleportation has been accomplished with two-mode squeezed states [10] and many other schemes have also been proposed such as light beams [11–15], atomic ensembles [16] and cold atoms [17]. It is well-known that CV systems embodied by light field modes may serve as reliable, long-haul information carriers, due to properties of robustness against decoherence [18–21].

On the other hand, atomic coherence, as a basic feature of quantum systems, has attracted much attention in the research of quantum optics in past decades. It can lead to various effects, such as electromagnetically induced transparency (EIT) [22], correlated spontaneous emission laser [23], change of spectra [24], population trapping [25], and a laser without inversion [26,27]. Recently, two important concepts, entanglement and atomic coherence, are shown to be closely related. Xiong et al. [28] and Tan

et al. [29] showed atomic coherence can generate fields that are entangled in a two-mode three-level cascade atomic system. Li et al. [30] proposed a scheme that coherence induced entanglement between two thermal fields of a three-level atom in V configuration. Wang et al. [31] proposed a scheme that coherence-enhanced and -controlled entanglement of two atoms in a single-mode thermal field.

In this paper, we will further discuss the relation between atomic coherence and entanglement in a two-mode three-level cascade atomic system. For the sake of comparison with the scheme discussed in references [28,29], we consider a three-level cascade atomic system that two photons of a strong external pump field induce atomic coherence by coupling the top and bottom levels. We show that the intracavity and output entanglement in the steady state can be generated by the external pump field under certain conditions.

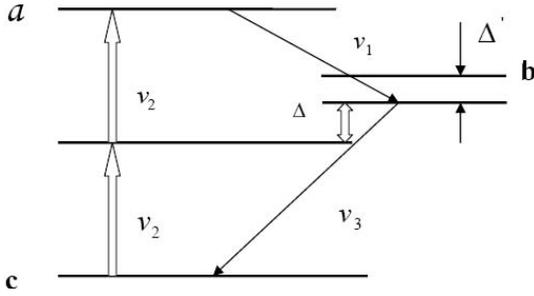
The organization of this paper is as follows. Section 2 gives the master equation for the two-mode field in the cavity. Section 3 is devoted to the entanglement analysis between the two modes in the cavity, and the entanglement of the output field is evaluated in this section. In Section 4, we present a summary of our main results.

## 2 The model

We consider a two-photon three-level cascade configuration as shown in Figure 1. The upper level  $a$  and the

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<sup>a</sup> e-mail: xyping@mail.hust.edu.cn



**Fig. 1.** Systematic diagram for a three-level atomic system in cascade configuration.

bottom level  $c$  have the same parity, but the intermediate level  $b$  has an opposite one. The dipole-allowed transitions  $a \leftrightarrow b$  and  $b \leftrightarrow c$  with frequencies  $\nu_1$  and  $\nu_3$ , respectively, are considered weak and treated quantum mechanically up to second order in coupling constant. The transition  $a \leftrightarrow c$  requires two pump photons of frequency  $\nu_2$ . Strong pump field is treated classically up to all orders. We assume that the one-photon pump detuning  $\omega_{bc} - \nu_2$  is sufficiently large that the dipole transition  $c \leftrightarrow b$  with pump frequency  $\nu_2$  is negligible. The pump frequency  $\nu_2$  is exactly one-half the atomic transition frequency  $\omega_{ac}$  ( $\equiv \omega_a - \omega_c$ ). The side-mode frequencies  $\nu_1$  and  $\nu_3$  are assumed to satisfy the conservation condition  $\nu_1 + \nu_3 = 2\nu_2$ , which gives the relation between the side-mode detuning  $\Delta'$  and the beat frequency  $\Delta \equiv \nu_2 - \nu_1$  as  $\Delta' = (\omega_{bc} - \nu_2) - \Delta$ . The Hamiltonian for the atom-field system is [32]

$$H = H_0 + V, \quad (1)$$

where the unperturbed part of the Hamiltonian is

$$H_0 = \sum_{i=a,b,c} \hbar\omega_i |i\rangle\langle i| + \sum_{j=1}^3 \hbar\nu_j a_j^\dagger a_j, \quad (2)$$

and the perturbed part is

$$V = \sum_{j=1}^3 \hbar g_j a_j U_j \sigma_j^\dagger + H.c., \quad (3)$$

where  $a_1$  and  $a_3$  are the annihilation operators for the field modes 1 and 3,  $a_2$  is the effective two-photon annihilation operator for the pump mode,  $U_j = U_j(r)$  is the spatial mode factor for the  $j$ th field mode, and  $g_j$  is the corresponding atom-field coupling constant. The matrices  $\sigma_j^\dagger$  are

$$\sigma_1^\dagger = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \sigma_2^\dagger = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \sigma_3^\dagger = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}. \quad (4)$$

The time dependence of the atom-field density operator  $\rho_{a-f}$  can be obtained from the basic density operator equation of motion, as

$$\frac{d}{dt} \rho_{a-f} = -\frac{i}{\hbar} [H, \rho_{a-f}] + r, \quad (5)$$

where  $r$  denotes the relaxation processes. By considering the slowly varying field modes and taking traces over the atomic states, the density matrix equation of motion for the field modes as obtained in reference [32] is

$$\begin{aligned} \frac{d}{dt} \rho = & -A_1 (\rho a_1 a_1^\dagger - a_1^\dagger \rho a_1) - (B_1 + \kappa_1) (a_1^\dagger a_1 \rho - a_1 \rho a_1^\dagger) \\ & -A_3 (\rho a_3 a_3^\dagger - a_3^\dagger \rho a_3) - (B_3 + \kappa_3) (a_3^\dagger a_3 \rho - a_3 \rho a_3^\dagger) \\ & +C_3 (a_3^\dagger a_1^\dagger \rho - a_1^\dagger \rho a_3^\dagger) + D_1 (\rho a_3^\dagger a_1^\dagger - a_1^\dagger \rho a_3^\dagger) + H.c., \end{aligned} \quad (6)$$

with  $\kappa_j$  ( $j = 1, 3$ ) is the damping constant of each mode. Different coefficients are given by

$$A_1 = \frac{N g_1^2 \mathcal{D}_1}{1 + I_2^2} \frac{f_a + I_2^2 \mathcal{D}_3^* D_2 / 4T_1 T_2}{1 + I_2^2 \mathcal{D}_1 \mathcal{D}_3^* / 4T_1 T_2}, \quad (7)$$

$$B_1 = \frac{N g_1^2 \mathcal{D}_1}{1 + I_2^2} \frac{f_b}{1 + I_2^2 \mathcal{D}_1 \mathcal{D}_3^* / 4T_1 T_2}, \quad (8)$$

$$A_3 = \frac{N g_3^2 \mathcal{D}_3}{1 + I_2^2} \frac{f_b}{1 + I_2^2 \mathcal{D}_1^* \mathcal{D}_3 / 4T_1 T_2}, \quad (9)$$

$$B_3 = \frac{N g_3^2 \mathcal{D}_3}{1 + I_2^2} \frac{f_c - I_2^2 \mathcal{D}_1^* D_2 / 4T_1 T_2}{1 + I_2^2 \mathcal{D}_1^* \mathcal{D}_3 / 4T_1 T_2}, \quad (10)$$

$$C_3 = \frac{i N g_3^2 \mathcal{D}_3}{1 + I_2^2} \frac{I_2}{2(T_1 T_2)^{1/2}} \frac{-f_a \mathcal{D}_1^* + D_2}{1 + I_2^2 \mathcal{D}_1^* \mathcal{D}_3 / 4T_1 T_2} e^{-i\phi}, \quad (11)$$

$$D_1 = \frac{i N g_1^2 \mathcal{D}_1}{1 + I_2^2} \frac{I_2}{2(T_1 T_2)^{1/2}} \frac{f_c \mathcal{D}_3^* + D_2}{1 + I_2^2 \mathcal{D}_1 \mathcal{D}_3^* / 4T_1 T_2} e^{-i\phi}. \quad (12)$$

The complex Lorentzian for the field modes 1 and 3 is

$$\mathcal{D}_{1,3} = \frac{1}{\gamma_{1,3} + i\Delta_{1,3}}, \quad (13)$$

where  $\gamma_1$  and  $\gamma_3$  are the dipole decay constants for  $a \leftrightarrow b$  and  $b \leftrightarrow c$  transitions,  $\Delta_1 = -\Delta_3 = -\Delta'$ .  $D_2 = 1/\gamma_2$ , where  $\gamma_2$  ( $\equiv 1/T_2$ ) is the two-photon coherent decay rate between the levels  $a$  and  $c$ . The dimensionless pump intensity  $I_2$  is defined by  $I_2 = 2|V_2|(T_1 T_2)^{1/2}$ , where  $V_2 = g_2 U_2 (n_2)^{1/2}$  is the effective two-photon interaction energy. The population difference decay time  $T_1$  is

$$T_1 = \frac{1}{\Gamma_a} \left[ 1 + \frac{\Gamma_1}{2\Gamma_3} \right]. \quad (14)$$

The probability factors  $f_k$  are

$$f_a = \frac{\Gamma_3}{\Gamma_1 + 2\Gamma_3} I_2^2, \quad (15)$$

$$f_b = \frac{\Gamma_1}{\Gamma_1 + 2\Gamma_3} I_2^2, \quad (16)$$

$$f_c = 1 + f_a. \quad (17)$$

Also  $\phi$  is the phase of the classical pump field and  $N$  is the total number of interacting atoms. The terms  $A_j$  and  $B_j$  with their complex conjugates are the gain and absorption coefficients for the  $j$ th mode, respectively, and  $C_3$  and  $D_1$  represent the coupling between the two modes.

### 3 Entanglement of the system

#### 3.1 Two-mode entanglement in the cavity

To determine the entanglement of the field modes, we need an entanglement criterion for the CV system. Here, we choose the summation of the quantum fluctuations proposed in reference [6]. According to this criterion, a state is entangled if the sum of the quantum fluctuations of the two Einstein-Podolsky-Rosen (EPR)-like operators  $\hat{u}$  and  $\hat{v}$  satisfies the following inequality

$$(\Delta\hat{u})^2 + (\Delta\hat{v})^2 < c^2 + \frac{1}{c^2}, \quad (18)$$

where  $c$  is an arbitrary (nonzero) real number,

$$\hat{u} = |c|\hat{x}_1 + \frac{1}{c}\hat{x}_2, \quad \hat{v} = |c|\hat{p}_1 - \frac{1}{c}\hat{p}_2, \quad (19)$$

with  $\hat{x}_1 = (a_1 + a_1^\dagger)/\sqrt{2}$ ,  $\hat{x}_2 = (a_3 + a_3^\dagger)/\sqrt{2}$ ,  $\hat{p}_1 = (a_1 - a_1^\dagger)/i\sqrt{2}$ , and  $\hat{p}_2 = (a_3 - a_3^\dagger)/i\sqrt{2}$  being the quadrature operators for the two modes 1 and 3. They satisfy the relation  $[\hat{x}, \hat{p}] = i\delta_{jj'}$  ( $j, j' = 1, 2$ ). For a general state, this is a sufficient criterion for entanglement, and for two-mode CV Gaussian states, this becomes a necessary and sufficient criterion. It has been shown that the state generated in a driven cascade three-level atomic system is a CV Gaussian state (in general a mixed state with the consideration of fluctuations) [29] and for such a state, this becomes a necessary and sufficient condition for entanglement.

The equations of motion for the first and second moments of the fields can be obtained from the master equation (6)

$$\frac{d}{dt}\langle a_1 \rangle = -\alpha_1\langle a_1 \rangle - D_1\langle a_3^\dagger \rangle, \quad (20)$$

$$\frac{d}{dt}\langle a_3 \rangle = -\alpha_3\langle a_3 \rangle + C_3\langle a_1^\dagger \rangle, \quad (21)$$

$$\frac{d}{dt}\langle a_1^\dagger a_1 \rangle = -\alpha_1\langle a_1^\dagger a_1 \rangle - D_1\langle a_3^\dagger a_1^\dagger \rangle + A_1 + c.c., \quad (22)$$

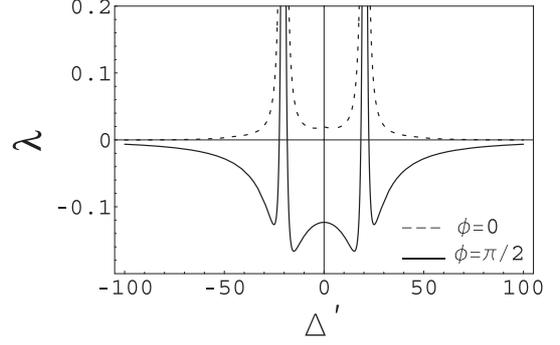
$$\frac{d}{dt}\langle a_3^\dagger a_3 \rangle = -\alpha_3\langle a_3^\dagger a_3 \rangle + C_3\langle a_1^\dagger a_1^\dagger \rangle + A_3 + c.c., \quad (23)$$

$$\frac{d}{dt}\langle a_1 a_3 \rangle = -(\alpha_1 + \alpha_3)\langle a_1 a_3 \rangle + C_3\langle a_1^\dagger a_1 \rangle - D_1\langle a_3^\dagger a_3 \rangle + C_3, \quad (24)$$

where  $\alpha_j = B_j - A_j + \kappa_j$  ( $j = 1, 3$ ). The steady-state solution of equations (20–24) can be found by setting  $d/dt = 0$ . Then we have  $\langle a_1 \rangle = \langle a_3 \rangle = 0$  and the second-order solutions can be expressed as follows

$$\begin{aligned} \langle a_1^\dagger a_1 \rangle &= \frac{1}{D} \{ A_1[\alpha_3(|\alpha_1 + \alpha_3|^2) + (\alpha_1 + \alpha_3)C_3D_1^* + c.c.] \\ &\quad + A_3(\alpha_1 + \alpha_3 + c.c.)|D_1|^2 - C_3D_1^*[(\alpha_3 + \alpha_3^*)(\alpha_1^* + \alpha_3^*) \\ &\quad + C_3D_1^* - C_3^*D_1] + c.c. \}, \quad (25) \end{aligned}$$

$$\begin{aligned} \langle a_3^\dagger a_3 \rangle &= \frac{1}{D} \{ A_3[\alpha_1(|\alpha_1 + \alpha_3|^2) + (\alpha_1 + \alpha_3)C_3^*D_1 + c.c.] \\ &\quad + A_1(\alpha_1 + \alpha_3 + c.c.)|C_3|^2 + |C_3|^2[(\alpha_1 + \alpha_1^*)(\alpha_1^* + \alpha_3^*) \\ &\quad + C_3^*D_1 - C_3D_1^*] + c.c. \}, \quad (26) \end{aligned}$$



**Fig. 2.** Variance  $\lambda$  vs.  $\Delta'$  for  $\phi = 0$  and  $\pi/2$ ,  $I_2 = 50$ ,  $C = 5$ ,  $\Gamma_a = 1$ ,  $\Gamma_1 = \Gamma_3 = 1$ , and  $\gamma_1 = \gamma_3 = \gamma_2 = 1$ . All frequencies are in units of  $\gamma_2$ .

$$\begin{aligned} \langle a_1 a_3 \rangle &= \\ &\frac{1}{D} \{ C_3(A_1 + A_1^*)[C_3D_1^* - C_3^*D_1 + (\alpha_3 + \alpha_3^*)(\alpha_1^* + \alpha_3^*)] \\ &\quad + (\alpha_1 + \alpha_1^*)[\alpha_1^*(\alpha_3 + \alpha_3^*)C_3 + C_3(C_3D_1^* + c.c.)] \\ &\quad + D_1(A_3 + A_3^*)[-C_3^*D_1 + C_3D_1^* - (\alpha_1 + \alpha_1^*)(\alpha_1^* + \alpha_3^*)] \\ &\quad + (\alpha_3 + \alpha_3^*)[\alpha_3^*(\alpha_1 + \alpha_1^*)C_3] \}. \quad (27) \end{aligned}$$

The denominator  $D$  in equations (25–27) is given by

$$\begin{aligned} D &= (\alpha_1 + \alpha_1^*)(\alpha_3 + \alpha_3^*)|\alpha_1 + \alpha_3|^2 \\ &\quad + (C_3^*D_1 - C_3D_1^*)^2 + \{ (\alpha_1 + \alpha_1^*)C_3D_1^*(\alpha_1 + \alpha_3) \\ &\quad + (\alpha_3 + \alpha_3^*)C_3^*D_1(\alpha_1 + \alpha_3) + c.c. \}. \quad (28) \end{aligned}$$

From them one can calculate the quantum fluctuations of the EPR operators  $\hat{u}$  and  $\hat{v}$ . The resulting expression is [33]

$$(\Delta\hat{u})^2 + (\Delta\hat{v})^2 = c^2 + \frac{1}{c^2} + 2\lambda, \quad (29)$$

with

$$\lambda = c^2\langle a_1^\dagger a_1 \rangle + \frac{1}{c^2}\langle a_3^\dagger a_3 \rangle + \frac{c}{|c|}\langle a_1 a_3 + a_1^\dagger a_3^\dagger \rangle, \quad (30)$$

where  $\langle a_j^\dagger a_j \rangle$  ( $j = 1, 3$ ) is the mean photon numbers in mode  $j$ ,  $c^2 = \sqrt{\langle a_3^\dagger a_3 \rangle / \langle a_1^\dagger a_1 \rangle}$ , and the sign of  $c$  is opposite to  $\langle a_1 a_3 + a_1^\dagger a_3^\dagger \rangle$ . By substituting equations (29, 30) into equation (18), we get a new criterion for the system: as long as the parameter

$$2\lambda = (\Delta\hat{u})^2 + (\Delta\hat{v})^2 - (c^2 + \frac{1}{c^2}) < 0, \quad (31)$$

one can confirm that the two-mode cavity fields are entangled.

In Figure 2 we have plotted  $\lambda$  against dimensionless detuning  $\Delta'$  for the phase choice of the pump field  $\phi = \pi/2$  and  $\phi = 0$ , respectively,  $I_2 = 50$  and the cooperativity parameter  $C = Ng^2/2\kappa\gamma = 5$  (for simplicity, we assume  $g_1 = g_2 = g$  and  $\kappa_1 = \kappa_3 = \kappa$ ). It is evident from the figure that the quantity  $\lambda$  is less than 0 for  $\phi = \pi/2$ , i.e., the two-mode entanglement is generated, while for  $\phi = 0$  no

entangled state can be built. In Figure 3 we have plotted  $\lambda$  versus  $I_2$  for zero detuning,  $\phi = \pi/2$ ,  $C = 1, 5, 10$ . It is clear that, for the weak pump intensity, the entanglement criterion (31) is not satisfied and no entangled state exists in the system. For certain large values of  $I_2$ , the entanglement for two-mode field in the cavity increases with the increasing of  $I_2$ , and has a maximum value and then it starts decreasing for further increasing value of  $I_2$ . With the increasing of  $C$ , the two-mode entanglement is significantly enhanced. In order to further see the dependence of entanglement on the dimensionless pump intensity  $I_2$  and the side-mode detuning  $\Delta'$ , in Figure 4 we have plotted  $\lambda$  as a function of  $\Delta'$  and  $I_2$  when  $C = 1, 5, 10$  and  $\phi = \pi/2$ . These figures show the region of  $\Delta'$  where we can get the entangled or disentangled states. Thus in this manner we can predict that the system exhibits two-mode entanglement in the cavity for the particular choice of the pump intensity, the phase of the intense driving field and the side-mode detuning.

### 3.2 Two-mode entanglement of the output field

Now, we will concentrate on the entanglement properties of the outgoing cavity field. The output field that has been considered for diverse applications can be measured through standard optical procedures [17,20]. By means of the input-output theory [34], the steady-state expressions for the spectral density of the second-order moments outside the cavity can be calculated along the same lines as discussed by Holm et al. [35] and An et al. [36]. The resulting expressions are

$$\langle a_1^\dagger a_1 \rangle_{out} = \frac{2\kappa(\alpha_3 - i\omega)(\alpha_3^* + i\omega)A_1 + |D_1|^2 A_3 - (\alpha_3^* + i\omega)D_1^* C_3 + c.c.}{|(\alpha_1 + i\omega)(\alpha_3^* + i\omega) + D_1 C_3^*|^2}, \quad (32)$$

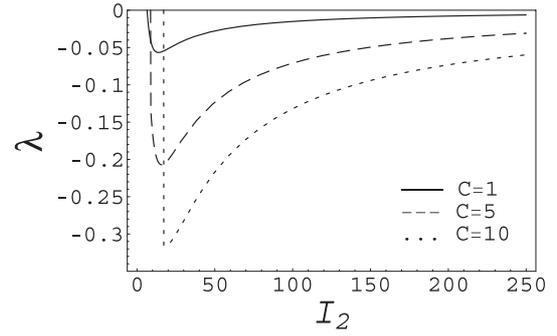
$$\langle a_3^\dagger a_3 \rangle_{out} = \frac{2\kappa(\alpha_1 - i\omega)(\alpha_1^* + i\omega)A_3 + |C_3|^2 A_1 + (\alpha_1^* + i\omega)C_3^* C_3 + c.c.}{|(\alpha_3 + i\omega)(\alpha_1^* + i\omega) + C_3 D_1^*|^2}, \quad (33)$$

$$\langle a_1 a_3 \rangle_{out} = \frac{2\kappa(\alpha_3^* + i\omega)C_3(A_1 + A_1^*) - (\alpha_1^* - i\omega)D_1(A_3 + A_3^*)}{|(\alpha_1 + i\omega)(\alpha_3^* + i\omega) + D_1 C_3^*|^2} + \frac{2\kappa(\alpha_1^* - i\omega)(\alpha_3^* + i\omega)C_3 - D_1 |C_3|^2}{|(\alpha_1 + i\omega)(\alpha_3^* + i\omega) + D_1 C_3^*|^2}, \quad (34)$$

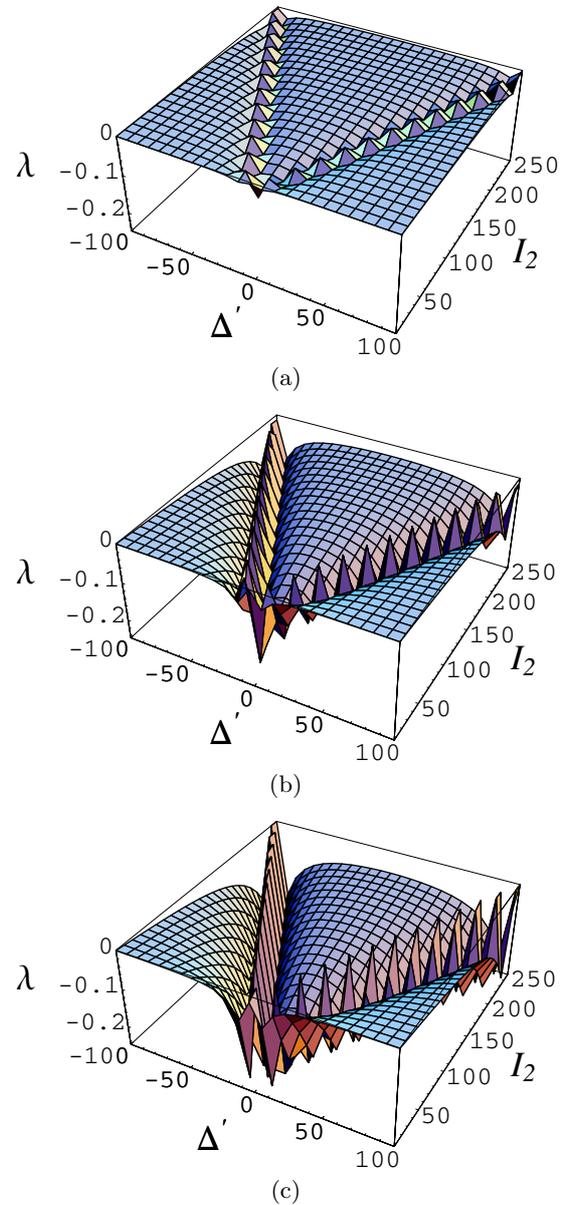
where  $\alpha_j = B_j - A_j + \kappa$  ( $j = 1, 3$ ) and  $\omega$  is the frequency deviation from the central frequency. We then have, for the output field,

$$\lambda' = c^2 \langle a_1^\dagger a_1 \rangle_{out} + \frac{1}{c^2} \langle a_3^\dagger a_3 \rangle_{out} + \frac{c}{|c|} \langle a_1 a_3 + a_1^\dagger a_3^\dagger \rangle_{out}. \quad (35)$$

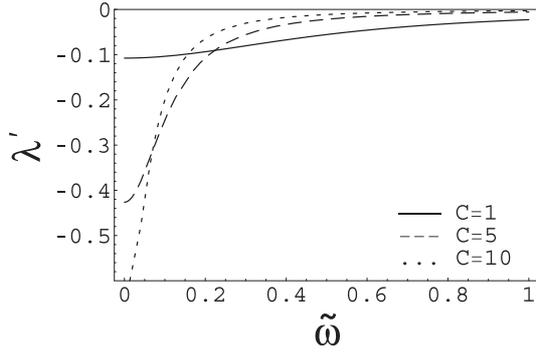
According to the discussion in Section 3.1, we know for the output field, as long as the parameter  $\lambda'$  is less than 0, one can confirm that entanglement of the output fields exists.



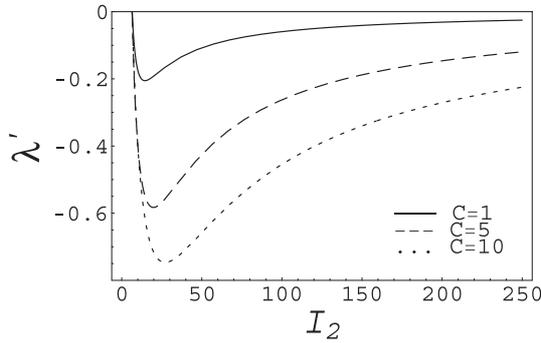
**Fig. 3.** Variance  $\lambda$  vs.  $I_2$  for  $\phi = \pi/2$ , and  $C = 1, 5$  and  $10$ ; the other parameters are the same as in Figure 2.



**Fig. 4.** Variance  $\lambda$  vs.  $\Delta'$  and  $I_2$  for  $\phi = \pi/2$ , and (a)  $C = 1$ , (b)  $C = 5$  and (c)  $C = 10$ ; the other parameters are the same as in Figure 2.



**Fig. 5.** Variance  $\lambda'$  vs.  $\tilde{\omega}$  for  $\phi = \pi/2$ , and  $C = 1, 5$  and  $10$ ; the other parameters are the same as in Figure 2.



**Fig. 6.** Variance  $\lambda'$  vs.  $I_2$  for  $\tilde{\omega} = 0$ ,  $\phi = \pi/2$ , and  $C = 1, 5$  and  $10$ ; the other parameters are the same as in Figure 2.

In Figure 5, we have plotted the dependence of  $\lambda'$  on the dimensionless frequency  $\tilde{\omega}$  for  $\phi = \pi/2$  and  $C = 1, 5, 10$ . It is clear that the entanglement degree gets its maximum at  $\tilde{\omega} = 0$  and decreases with the increasing of  $\tilde{\omega}$ , i.e., better entanglement of the output field can be generated at the central frequency ( $\tilde{\omega} = 0$ ). In Figure 6, we have plotted  $\lambda'$  versus dimensionless pump intensity  $I_2$ , for  $\tilde{\omega} = 0$ ,  $\Delta' = 0$ ,  $\phi = \pi/2$  and  $C = 1, 5, 10$ . Comparing with the intracavity entanglement, the similar entanglement characteristics can be obtained outside the cavity. Physically we can understand these results by noting the following point: the spontaneous-emission processes may take an important effect on the disentanglement for the low-pump intensity. A lack of spontaneous emission aids in the generation of good entanglement. On the other hand, for strong pump intensity, the two-mode entanglement can be achievable. This is due to the Rabi splitting of the upper level  $a$  accompanied by vanishing splitting level  $b$  which leads to negligible spontaneous emission. In fact, better entangled states can be generated by choosing suitable values of the cooperativity parameter and pump intensity which may be approached in experiment.

## 4 Conclusion

In summary, the generation of continuous variable entanglement via atomic coherence in a two-mode three-level cascade atomic system is investigated. Atomic coherence

between the top and bottom levels is induced with two photons of a strong external pump field. It shows that the intracavity and output entanglement in the steady state can be achievable under some conditions. It is also found that the two-mode entanglement shows some strong dependence on the intensity of the pump field, the side-mode detuning and the values of cooperativity parameter. The generation of two-mode entangled states has been proposed and experimentally implemented [15,37]. Our scheme indicates that such two-mode entangled states can be implemented on the basis of microwave cavity-quantum electrodynamics [38]. We predict that the results in this paper, based on a three-level atomic system including cavity losses and spontaneous emission, can be verified by an experiment such as the one used in reference [39].

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## References

1. C.H. Bennett et al., Phys. Rev. Lett. **70**, 1895 (1993)
2. A. Ekert, R. Jozsa, Rev. Mod. Phys. **68**, 733 (1996); E. Knill, R. Laflamme, G.J. Milburn, Nature **409**, 46 (2001)
3. A.K. Ekert, Phys. Rev. Lett. **67**, 661 (1991)
4. S.L. Braunstein, P.V. Loock, Rev. Mod. Phys. **77**, 513 (2005)
5. R. Simon, Phys. Rev. Lett. **84**, 2726 (2000)
6. L.M. Duan, G. Giedke, J.I. Cirac, P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000)
7. M. Hillery, M.S. Zubiry, Phys. Rev. Lett. **96**, 050503 (2006)
8. P. Hyllus, J. Eisert, New J. Phys. **8**, 51 (2005)
9. E. Shchukin, W. Vogel, Phys. Rev. Lett. **95**, 230502 (2005)
10. A. Furusawa, J.L. Sorensen, S.L. Braunstein, C.A. Fuchs, H.J. Kimble, E.S. Polzik, Science **282**, 706 (1998); S.L. Braunstein, H.J. Kimble, Phys. Rev. Lett. **80**, 869 (1998)
11. Z.Y. Ou, S.F. Pereira, H.J. Kimble, K.C. Peng, Phys. Rev. Lett. **68**, 3663 (1992)
12. W.P. Bowen, R. Schnabel, P.K. Lam, T.C. Ralph, Phys. Rev. Lett. **90**, 043601 (2003)
13. C. Schori, J.L. Sorensen, E.S. Polzik, Phys. Rev. A **66**, 033802 (2002)
14. X. Jia, X. Su, Q. Pan, J.R. Gao, C.D. Xie, K. Peng, Phys. Rev. Lett. **93**, 250503 (2004)
15. A.S. Villar, L.S. Cruz, K.N. Cassemiro, M. Martinelli, P. Nussenzveig, Phys. Rev. Lett. **95**, 243603 (2005)
16. B. Julsgaard, A. Kozhekin, E.S. Polzik, Nature **413**, 400 (2001)
17. V. Josse, A. Dantan, A. Bramati, M. Pinard, E. Giacobino, Phys. Rev. Lett. **92**, 123601 (2004)
18. S. Olivares, M.G.A. Paris, J. Opt. B: Quantum Semiclass. Opt. **6**, 69 (2004); H. Jeong, J. Lee, M.S. Kim, Phys. Rev. A **61**, 052101 (2000); J. Lee, M.S. Kim, H. Jeong, Phys. Rev. A **62**, 032305 (2000)
19. P. Marek, M.S. Kim, M. Paternostro, Phys. Rev. A **74**, 032311 (2006)
20. R. Guzman, J.C. Retamal, E. Solano, N. Zagury, Phys. Rev. Lett. **96**, 010502 (2006)

21. S. Mancini, V. Giovannetti, D. Vitali, P. Tombesi, *Phys. Rev. Lett.* **88**, 120401 (2002); S. Mancini, D. Vitali, V. Giovannetti, P. Tombesi, *Eur. Phys. J. D* **22**, 417 (2003)
22. S.E. Harris et al., *Phys. Rev. Lett.* **64**, 1498 (1990); K.J. Boller et al., *Phys. Rev. Lett.* **66**, 1033 (1991); S. Jin et al., *Opt. Commun.* **119**, 90 (1995)
23. M.O. Scully, *Phys. Rev. Lett.* **55**, 2802 (1985); M.O. Scully, M.S. Zubairy, *Phys. Rev. A* **35**, 752 (1987)
24. P. Zhou, S. Swain, *Phys. Rev. Lett.* **78**, 832 (1997); S.Y. Zhu et al., *Phys. Rev. A* **52**, 4791 (1995)
25. O.A. Kocharovskaya et al., *Opt. Commun.* **84**, 393 (1991)
26. M.O. Scully et al., *Phys. Rev. Lett.* **62**, 2813 (1989); S.E. Harris, *Phys. Rev. Lett.* **62**, 1033 (1989)
27. X.M. Hu, J.S. Peng, *Eur. Phys. J. D* **5**, 291 (1999)
28. H. Xiong, M.O. Scully, M.S. Zubairy, *Phys. Rev. Lett.* **94**, 023601 (2005)
29. H.T. Tan, S.Y. Zhu, M.S. Zubairy, *Phys. Rev. A* **72**, 022305 (2005)
30. F. Li, H. Xiong, M.S. Zubairy, *Phys. Rev. A* **72**, 010303 (2005)
31. X. Wang, S. Du, X. Chen, *J. Phys. B: At. Mol. Opt. Phys.* **39**, 3805 (2006)
32. N.A. Ansari, *Phys. Rev. A* **46**, 1560 (1992)
33. Y.J. Xia, G.C. Guo, *Chin. Phys. Lett.* **21**, 1877 (2004)
34. M.J. Collet, C.W. Gardiner, *Phys. Rev. A* **30**, 1386 (1984)
35. D.A. Holm, M. Sargent III, *Phys. Rev. A* **35**, 2150 (1987)
36. S. An, M. Sargent, *Phys. Rev. A* **39**, 1841 (1989)
37. W.P. Bowen, N. Treps, R. Schnabel, P.K. Lam, *Phys. Rev. Lett.* **89**, 053601 (2002); J. Laurat et al., *J. Opt. B: Quantum Semiclass. Opt.* **7**, 577 (2005)
38. J.M. Raimond, M. Brune, S. Haroche, *Rev. Mod. Phys.* **73**, 565 (2001)
39. C.H. van der Wal, M.D. Eisaman, A. Andre, R.L. Walsworth, D.F. Phillips, A.S. Zibrov, M.D. Lukin, *Science* **301**, 196 (2003)